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# THE SYNTHESIS OF METEOROID DISTRIBUTIONS FROM MONOENERGETIC MONODIRECTIONAL KERNELS

*by R. D. Shelton, H. E. Stern, J. J. Wright, and D. P. Hale*

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# THE SYNTHESIS OF METEOROID DISTRIBUTIONS FROM MONOENERGETIC MONODIRECTIONAL KERNELS

## SUMMARY

As a logical extension from previous papers by the authors, a general method for calculation of the meteoric flux at any finite point, given an arbitrary spectrum at infinity of broad meteoric streams, is derived. This formalism is explicitly applied, obtaining plots of relative flux versus angle for a series of examples dealing with monoenergetic isotropic distributions, limited monoenergetic isotropic distributions and transformed isotropic cases -- that is, apparent flux as seen about an attractive center (for example, Earth) moving relative to a system in which the distribution is isotropic at infinity. A mathematical appendix outlining the computational methods employed is included.

## SECTION I. INTRODUCTION

The effect of a gravitational center on an infinite parallel stream and an isotropic distribution of meteoroids has been studied in detail by Shelton, Stern, and Hale [1], and by Hale and Wright [2, 3]. The purpose of this study is to investigate a process by which flux patterns obtained from the parallel stream treatment can be superimposed to approximate any arbitrary distribution of micrometeoroids in speed and direction, and to apply this process to special cases to illustrate the behavior of possible meteoroid distributions in the vicinity of the Earth.

Define a velocity space wherein each particle represents a meteoric stream of unit flux at infinity. Each stream need not be of infinite extent or persist eternally; however, it is convenient to regard these streams as being of such breadth and location to include both the origin (that is, the center) and some common point $\vec{r}_o$ sufficiently far from the origin that the flux patterns there are essentially undisturbed by the attractive center.

At $\vec{r}_o$ let the streams be specified by a density $N(v_\infty, \Theta, \Phi)$ in a stream velocity space, where $v_\infty$ is the speed of the stream at infinity (and approximately the speed at $\vec{r}_o$); $\Theta$ and $\Phi$ are the usual angular coordinates which, along with

$v_\infty$, define a velocity vector. However, one should note that $\Theta$ and $\Phi$ define the direction from which the stream comes. Within an arbitrarily small element of solid angle, $\sin \Theta \, \Delta\Theta \, \Delta\Phi$ in velocity space, the number of almost parallel streams of speeds between $v_\infty$ and $v_\infty + \Delta v_\infty$ is

$$N(v_\infty, \Theta, \Phi) \, v_\infty^2 \, \sin \Theta \, \Delta\Theta \, \Delta\Phi \, \Delta v_\infty ,$$

and the flux at $\vec{r}_o$ due to these streams is

$$v_\infty \cdot N(v_\infty, \Theta, \Phi) \, v_\infty^2 \, \sin \Theta \, \Delta\Theta \, \Delta\Phi \, \Delta v_\infty .$$

These streams at any other point $(r, \theta')$, where $r << r_o$ and $\theta'$ is defined to be zero in the direction $(\Theta, \Phi)$, will give rise to a flux

$$f(v_\infty, r, \theta') \, N(v_\infty, \Theta, \Phi) \, v_\infty^3 \, \sin \Theta \, \Delta\Theta \, \Delta\Phi \, \Delta v_\infty ,$$

where $f(v_\infty, r, \theta')$ is the flux field pattern for a unit flux at infinity derived in the first three references.

Generally, there may be many streams simultaneously active in different directions. For each infinitely broad stream, one can define a $\vec{r}_o$ in terms of $(\Theta, \Phi)$, the stream's radiant. $\vec{r}_o$ is simply defined as a point at a great distance in the real space direction $(\Theta, \Phi)$. The total flux F at a point $(r, \theta, \phi)$ can thus be formally expressed as

$$F(r, \theta, \phi) = \int_{v_\infty} \int_{\Theta} \int_{\Phi} f(v_\infty, r, \theta') \, v_\infty^3 \, N(v_\infty, \Theta, \Phi) \sin \Theta \, d\Theta \, d\Phi \, dv_\infty , \quad (1)$$

where $\theta'$ is the angle between the directions $(\Theta, \Phi)$ and $(\theta, \phi)$ by definition.

This formalism can be applied to streams of finite breadth (pencils), and all of the pattern details for any one stream as previously developed [1, 2, 3] including isoflux and isodensity contours will be valid, provided that the region of interest lies within the volume swept through by the pencils. Of course at the boundary of a pencil, or finite stream, its contribution to the total flux and density goes abruptly to zero.

A uniform monoenergetic particle stream monodirectional at infinity, flowing by a disturbing gravitational center, develops a flux pattern which is axially symmetric about a line through the gravitational center and in the original stream direction. The flux pattern has been expressed [1,2,3] in the form $f(v_\infty, r, \theta')$, where r is the distance from the origin, and the angle $\theta'$ is measured from the upstream symmetry axis.

## SECTION II. THE EXPLICIT FORM OF $f(v_\infty, r, \theta')$

Consider a parallel stream from the direction defined by $\Theta$ and $\Phi$, as shown in Figure 1, and an arbitrary point P at which the particle flux is to be determined. Because of the axial symmetry of the stream disturbance, the flux at P depends only on the distance from the origin and the angle $\theta'$. The cosine of $\theta'$ can be computed by taking the scalar product of two unit vectors, one along the symmetry axis and the other in the direction of the radius vector drawn from the origin to the point P. Thus,

$$\cos\theta' = \hat{k}' \cdot \hat{r}\ , \tag{2}$$

where

$$\hat{k}' \equiv \hat{i}\sin\Theta\cos\Phi + \hat{j}\sin\Theta\sin\Phi + \hat{k}\cos\Theta$$
$$\hat{r} \equiv \hat{i}\sin\theta\cos\phi + \hat{j}\sin\theta\sin\phi + \hat{k}\cos\theta,$$

or

$$\cos\theta' = \sin\Theta\sin\theta\cos(\Phi - \phi) + \cos\Theta\cos\theta. \tag{3}$$

In equations 2 and 3, $\hat{i}$, $\hat{j}$, $\hat{k}$ are unit vectors along the x, y, z axes, respectively.

Imagine, as in Figure 2, a parallel stream of meteoroids approaching a gravitational center E. If the flux in the undisturbed stream is unity, the number of particles per second crossing an annular strip between a and a + $\Delta$a, where a is the impact parameter, is given by $2\pi a \Delta a$. If these particles impact on a sphere of radius r, they will fall on a zone whose area is $2\pi r^2 \sin\theta' \Delta\theta'$. Since particles are conserved, the rate at which particles cross the two surfaces are equal, and we can write:

$$2\pi a \Delta a = f(v_\infty, r, \theta')\, 2\pi r^2 \sin\theta' \cos\alpha\, \Delta\theta'\ , \tag{4}$$

where $f(v_\infty, r, \theta')$ is the flux at the spherical surface and $\alpha$ is the angle between the inward radius vector and particle direction measured positive counterclockwise. Solving for $f(v_\infty, r, \theta')$, we have

$$f(v_\infty, r, \theta') = \frac{a}{r^2 \sin\theta' \cos\alpha} \left(\frac{\partial a}{\partial \theta'}\right)_r\ , \tag{5}$$

where we have gone to differential form on the basis of the relationship between a, the impact parameter, and the variables describing the motion of the particles in a gravitational field. It is possible, in an analogous manner using a conical zone (that is, surface

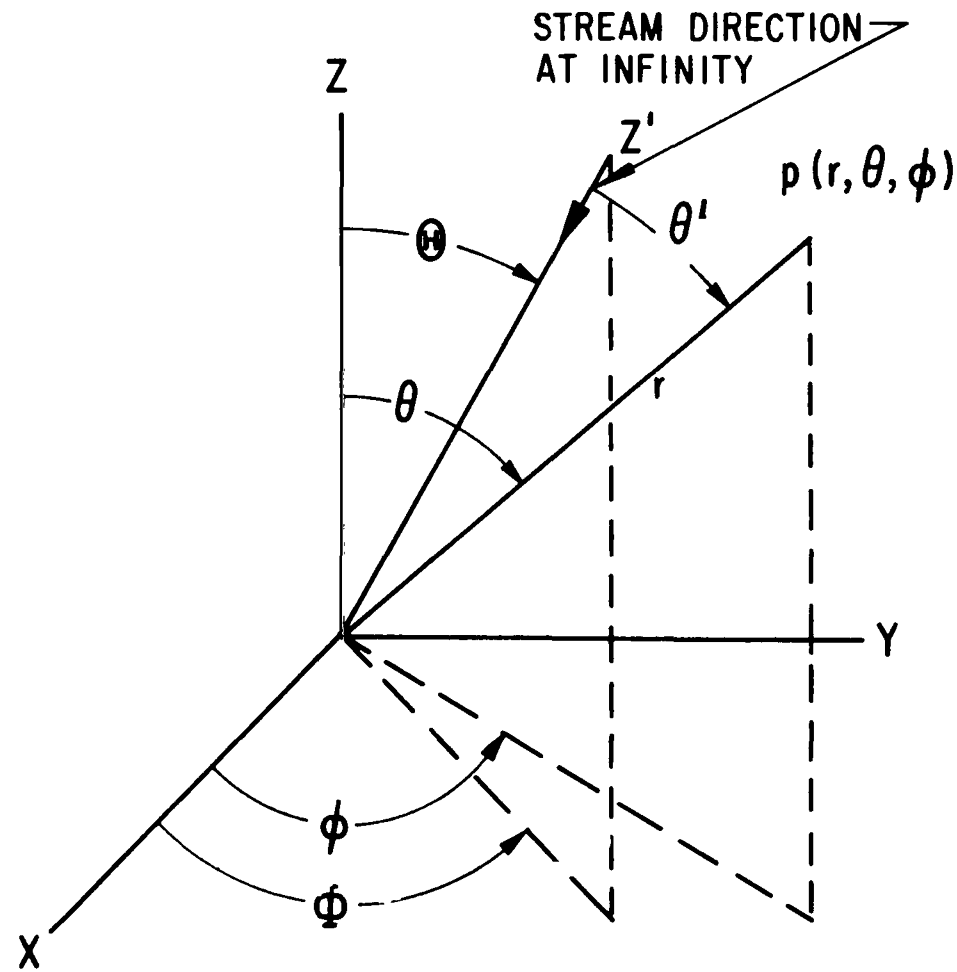


FIGURE 1. GEOMETRY FOR THE FLUX CALCULATIONS. P IS THE POINT, LOCATED BY r, $\theta$, AND $\phi$, AT WHICH THE FLUX (particles/$m^2$ sec) IS DESIRED.

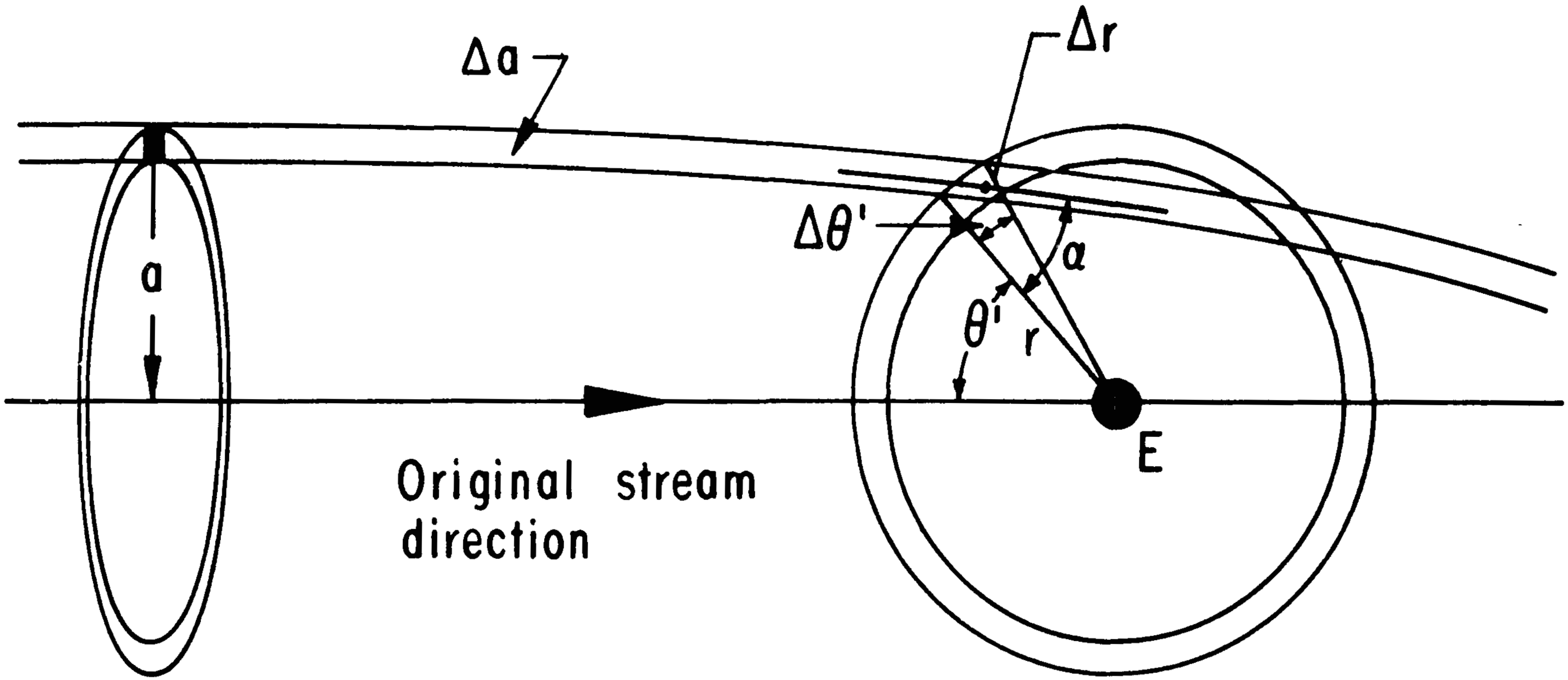


FIGURE 2. GEOMETRY FOR PARALLEL STREAM FLUX PATTERN CALCULATION

defined by $\theta'$ = constant, r = r and r = r + $\Delta$r), to express the flux pattern function as

$$f(v_\infty, r, \theta') = \frac{a}{r \sin \theta' \sin \alpha} \left(\frac{\partial a}{\partial r}\right)_{\theta'} \tag{6}$$

The simple situation of Figure 2 is complicated by the fact that the flux at a point may be composed of two or more streams or be intercepted by a finite gravitational center [3].

Following the work of Hale and Wright [2, 3] and noting equation 5, we write

$$f(v_\infty, r, \theta') = \left| \frac{a_+}{r^2 \sin \theta' \cos \alpha_+} \left(\frac{\partial a_+}{\partial \theta'}\right)_r \right| + \left| \frac{a_-}{r^2 \sin \theta' \cos \alpha_-} \left(\frac{\partial a_-}{\partial \theta'}\right)_r \right| , \tag{7}$$

where the first term is direct flux (approaching perigee), the second is scattered flux (receding from perigee), and

$$a_\pm = \left(\frac{1}{2y}\right) \left[ ry \sin \theta' \pm \sqrt{r^2 y^2 \sin^2 \theta' + 4 yr (1-\cos \theta')} \right] \tag{8}$$

$$\left(\frac{\partial a_\pm}{\partial \theta'}\right)_r = \left(\frac{1}{2}\right) \left\{ r \cos \theta' \pm \frac{r^2 y \cos \theta' \sin \theta' + 2r \sin \theta'}{[r^2 y^2 \sin^2 \theta' + 4 yr (1-\cos \theta')]^{\frac{1}{2}}} \right\} \tag{9}$$

$$y \equiv \frac{v_\infty^2}{\gamma M} \tag{10}$$

$$\cos \alpha_\pm = \pm \left[ \frac{r^2 y^2 + 2 yr - y^2 a_\pm^2}{r^2 y^2 + 2 ry} \right]^{\frac{1}{2}} , \tag{11}$$

where $v_\infty$ is the speed of the stream at large distances, $\gamma$ is the gravitational constant, and M is the mass of the gravitational center. Either of the two terms in equation 7 may vanish if the stream intersects the Earth before arriving at the point (r, $\theta$, $\phi$). From simple angular momentum considerations [1, 2, 3] one can show that the first term of equation 7 is nonzero if either of the following conditions is satisfied:

$$a_+ > R \; [1 + 2/yR]^{\frac{1}{2}} \tag{12}$$

or

$$0 < \theta' < \theta'_k , \tag{13}$$

where

$$\theta'_k \equiv \cos^{-1}\left[-1/(1+yR)\right]$$

and R is the radius of the finite attractive center.

The second term of equation 7 is nonzero only if

$$|a_-| > R\left[1 + 2/yR\right]^{\frac{1}{2}} \tag{14}$$

SECTION III. SINGULARITIES IN $f(v_\infty, r, \theta')$

The denominators of the terms in equation 7 may go to zero as a result of geometric properties despite the well-behaved nature of the physical process. In particular, we know that $f(v_\infty, r, \theta')$ is well defined for $\theta'$ equal to zero and $\alpha_+$ equal to $\pi/2$. When $\theta'$ equals zero, the first term of equation 7 is replaced by

$$\lim_{\theta' \to 0} f(v_\infty, r, \theta') = \left[\frac{ry + \sqrt{r^2y^2 + 2\,yr}}{2\,yr}\right]^2 , \tag{15}$$

and the second term is removed by inequality of equation 14.

The singularity for $\alpha_+$ equal to $\pi/2$ is easily removed by using equation 6 rather equation 5. A tedious evaluation yields

$$\lim_{\alpha_+ \to \pi/2} \frac{a_+ \left(\partial a_+/\partial r\right)_{\theta'}}{r \sin\theta' \sin\alpha_+} = \frac{(ry + 1)^2}{ry\,(r^2y^2 + 2ry)^{\frac{1}{2}}} , \tag{16}$$

which is used for machine computations if $\cos\alpha_+$ becomes too small for accurate computation of $f(v_\infty, r, \theta')$ by equation 5.

The singularity at $\theta' = \pi$ has a strong physical basis. It can be shown, however, that the number of particles per unit time crossing a small disc located on and normal to the symmetry axis is finite and, in the limit as the radius of the disc approaches zero, is proportional to the radius. If the distribution in direction is continuous and finite rather than discrete, it follows that there can be no point singularities in the flux pattern. If the planets are viewed as scattering centers for the meteoroids, it is likely that the meteoroid distribution in velocity is piecewise continuous and that the unusual focussing effects demonstrated [2, 3] for broad persistent parallel streams are highly unlikely.

## SECTION IV. THE MONOENERGETIC ISOTROPIC DISTRIBUTION

If the meteoric streams are traveling in all directions with equal probability and all have the same speed $v_\infty$, the unnormalized density distribution function may be written as

$$N(v_\infty, \Theta, \Phi) = \delta(v_\infty - v'_\infty) \tag{17}$$

The particle flux F at infinity is therefore given by

$$F(\infty) = \iiint v'^3_\infty \, \delta(v_\infty - v'_\infty) \sin\Theta \, d\Theta \, d\Phi dv'_\infty = 4\pi v^3_\infty, \tag{18}$$

so that the normalized density distribution function (one that will yield unit flux) for the isotropic case is

$$N(v_\infty, \Theta, \Phi) = \delta(v_\infty - v'_\infty)/4\pi v^3_\infty. \tag{19}$$

The substitution of this distribution function into equation 1 yields

$$F(r, \theta, \phi) = \frac{1}{4\pi} \int_{\mu=-1}^{\mu=1} \int_{\Phi=0}^{\Phi=2\pi} f(v_\infty, r, \theta') \, d\mu \, d\Phi, \tag{20}$$

where use has been made of the definition

$$\mu = \cos\Theta$$

Since the isotropic case can be treated in closed form [1, 4], the evaluation of this integral provides a check on the machine computation and on the method of handling the singular point at $\theta' = \pi$. Table 1 shows a comparison between the machine computation of the integral of equation 20 and the results obtained by completely analytical methods [1].

## SECTION V. LIMITED MONOENERGETIC ISOTROPIC DISTRIBUTIONS

If the distribution in direction is isotropic except for some forbidden cones of directions, equation 20 may still be used after modifying the normalization factor and the limits of integration. If the meteoroid flux is restricted to a cone of directions such that if

$$\begin{cases} A \le \mu \le 1, \; N(v_\infty, \mu, \Phi) = k\,\delta(v_\infty - v'_\infty) \\ -1 \le \mu \le A, \; N = 0 \end{cases} \tag{21}$$

# TABLE I

## COMPARISON OF ISOTROPIC FLUX COMPUTED BY TWO METHODS

| $V_\infty = 10$ km/sec | | | | $V_\infty = 40$ km/sec | | | |
|---|---|---|---|---|---|---|---|
| $\frac{r}{R}$ | ANALYTIC | MACHINE COMPUTATION | | $\frac{r}{R}$ | ANALYTIC | MACHINE COMPUTATION | |
| | | detector at 30° | detector at 90° | | | detector at 30° | detector at 90° |
| 1.0 | 1.1130 | 1.1131 | 1.1132 | 1.0 | .5383 | .5387 | .5446 |
| 1.5 | 1.5220 | 1.5224 | 1.5164 | 1.5 | .9134 | .9121 | .9099 |
| 2.0 | 1.4592 | 1.4625 | 1.4558 | 2.0 | .9659 | .9664 | .9664 |
| 3.0 | 1.3438 | 1.3444 | 1.3407 | 3.0 | .9947 | .9935 | 9931 |
| 4.0 | 1.2707 | 1.2714 | 1.2680 | 4.0 | 1.0020 | 1.0023 | 1.0006 |
| 5.0 | 1.2225 | 1.2229 | 1.2201 | 5.0 | 1.0044 | 1.0040 | 1.0051 |
| 10.0 | 1.1170 | 1.1171 | 1.1155 | 10.0 | 1.0049 | 1.0048 | 1.0045 |

$R = 6.528 \times 10^6$ meters

k can be determined by substitution into equation 1 and setting $f(r, \theta, \phi) = 1$ for $r = \infty$ (that is, $f(v_\infty, \infty, \theta') = 1$). The normalized result is

$$N(v_\infty, \mu, \Phi) = \begin{cases} \dfrac{\delta(v_\infty - v'_\infty)}{2\pi(1-A)\, v_\infty^3}, & \text{if } A \leq \mu \leq 1 \\ 0, \text{ if } \mu < A < 1 \end{cases} \tag{22}$$

The integral of equation 1 becomes

$$F(r, \theta, \phi) = \frac{1}{2\pi(1-A)} \int_{\mu=A}^{\mu=1} \int_{\Phi=0}^{\Phi=2\pi} f(v_\infty, r, \theta')\, d\mu\, d\Phi \tag{23}$$

Figures 3 through 6 illustrate this for the limited polar isotropic case, for $A = 0$ ( that is, $0° \leq \Theta \leq 90°$ ), $v_\infty = 10$ km/sec; $A = 0$, $v_\infty = 40$ km/sec; $A = \sqrt{3}/2$ (that is, $0° \leq \Theta \leq 30°$), $v_\infty = 10$ km/sec; and $A = \sqrt{3}/2$, $v_\infty = 40$ km/sec. The cases show that the anistropy in the velocity distribution has introducted angular variations into the flux patterns.

If the velocity distribution in solid angle in velocity space is a spherical zone centered about the circle defined by $\Theta = \pi/2$ and bounded by $\Theta = \pi/2 \pm \cos^{-1} B$, the limited equatorial isotropic case, the normalized density distribution may be written as

$$N(v_\infty, \mu, \Phi) = \begin{cases} \dfrac{\delta(v_\infty - v'_\infty)}{4\pi B\, v_\infty^3}, & \text{if } -B \leq \mu \leq B \\ 0, \text{ if } \mu < -B \text{ or } \mu > B \end{cases} \tag{24}$$

and the flux integral becomes

$$F(r, \theta, \phi) = \frac{1}{4\pi B} \int_{\mu=-B}^{\mu=B} \int_{\Phi=0}^{\Phi=2\pi} f(v_\infty, r, \theta')\, d\mu\, d\Phi. \tag{25}$$

Figures 7 through 10 illustrate the flux distributions arising for $B = 0.5$ (that is, $60° \leq \Theta \leq 120°$ ), $v_\infty = 10$ km/sec; $B = 0.5$, $v_\infty = 40$ km/sec; $B = (2 - \sqrt{3})/4$ (that is, $86°.17 \leq \Theta \leq 93°.83$ ), $v_\infty = 10$ km/sec; $B = (2 - \sqrt{3})/4$, $v_\infty = 40$ km/sec. The solid angles subtended by the allowable directions are the same here as in the previous examples, (that is, equation 23) being $2\pi$ and $2\pi\ (1 - \sqrt{3}/2)$ for both examples. A comparison of the limited polar isotropic cases with their limited equatorial isotropic cases analogous (that is, same $v_\infty$ and same allowable magnitudes of solid angle for stream

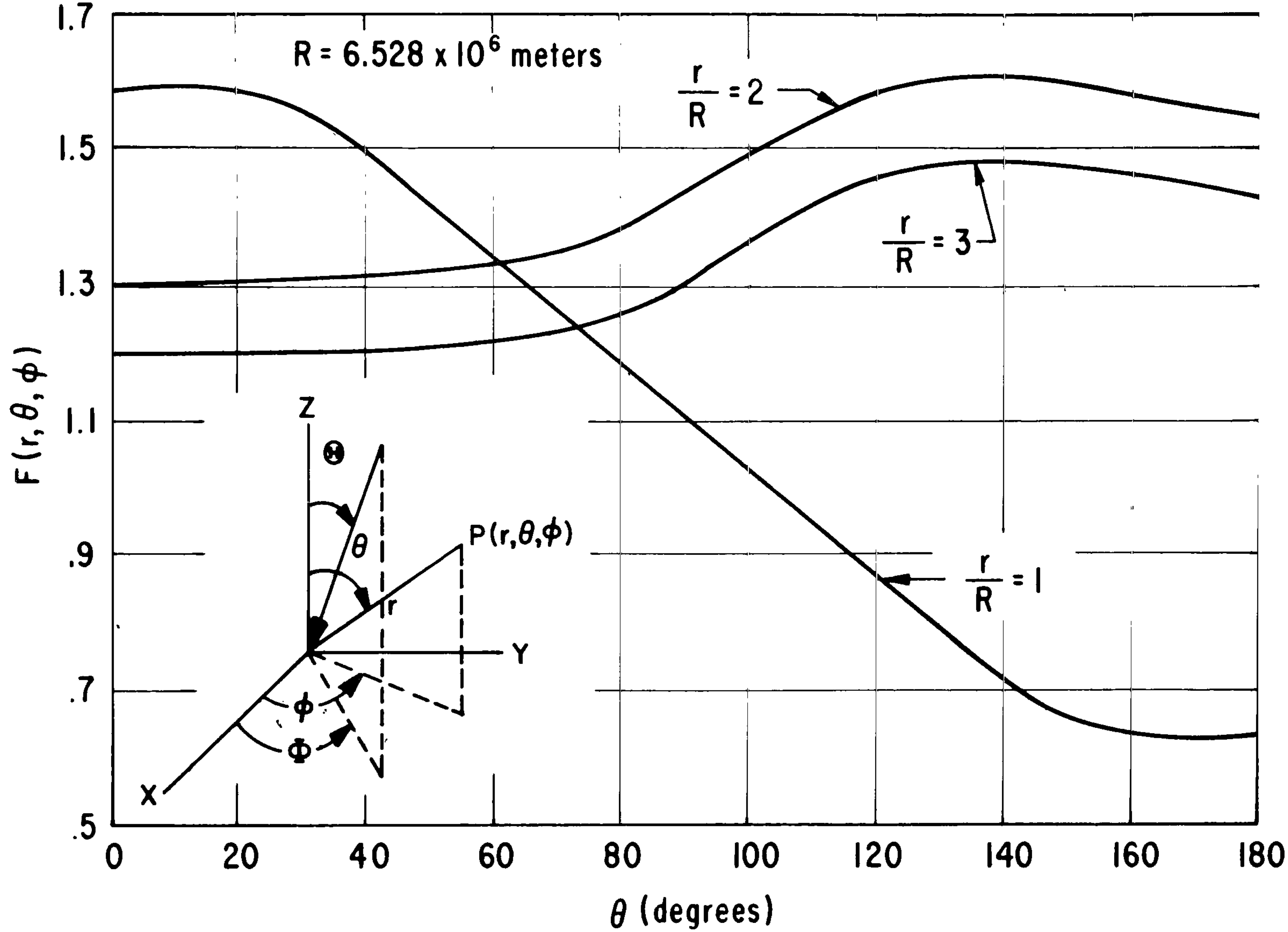


FIGURE 3. METEOROID FLUX FOR THE LIMITED POLAR ISOTROPIC CASE WITH $v_\infty = 10$ km/sec, $0° \leq \Theta \leq 90°$, $0° \leq \Phi \leq 360°$ AND $\phi = 0°$, ASSUMING UNIT FLUX AT INFINITY.

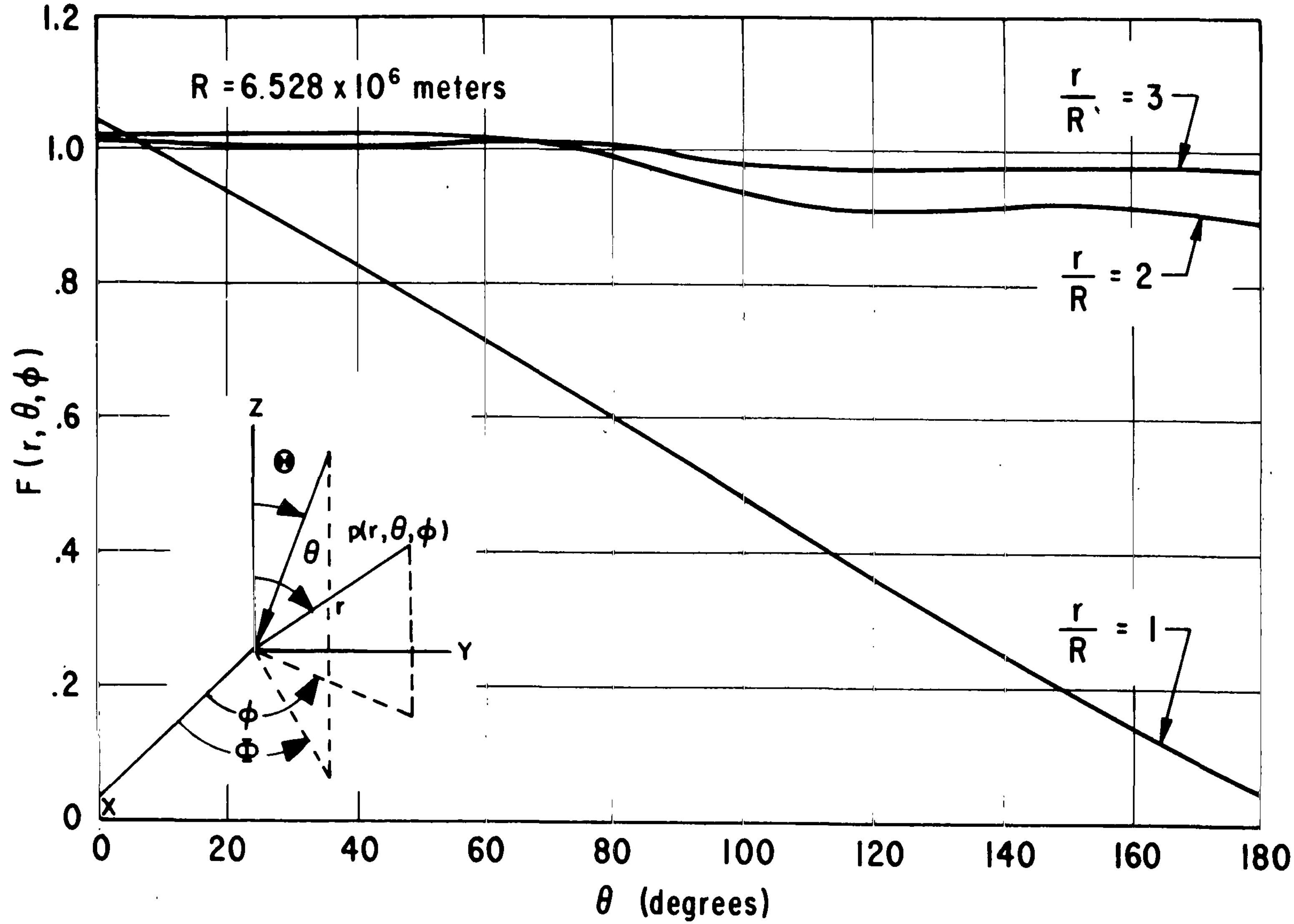


FIGURE 4. METEOROID FLUX FOR THE LIMITED POLAR ISOTROPIC CASE WITH $v_{\infty} = 40$ km/sec, $0° \leq \Theta \leq 90°$, $0° \leq \Phi \leq 360°$ AND $\phi = 0°$, ASSUMING UNIT FLUX AT INFINITY.

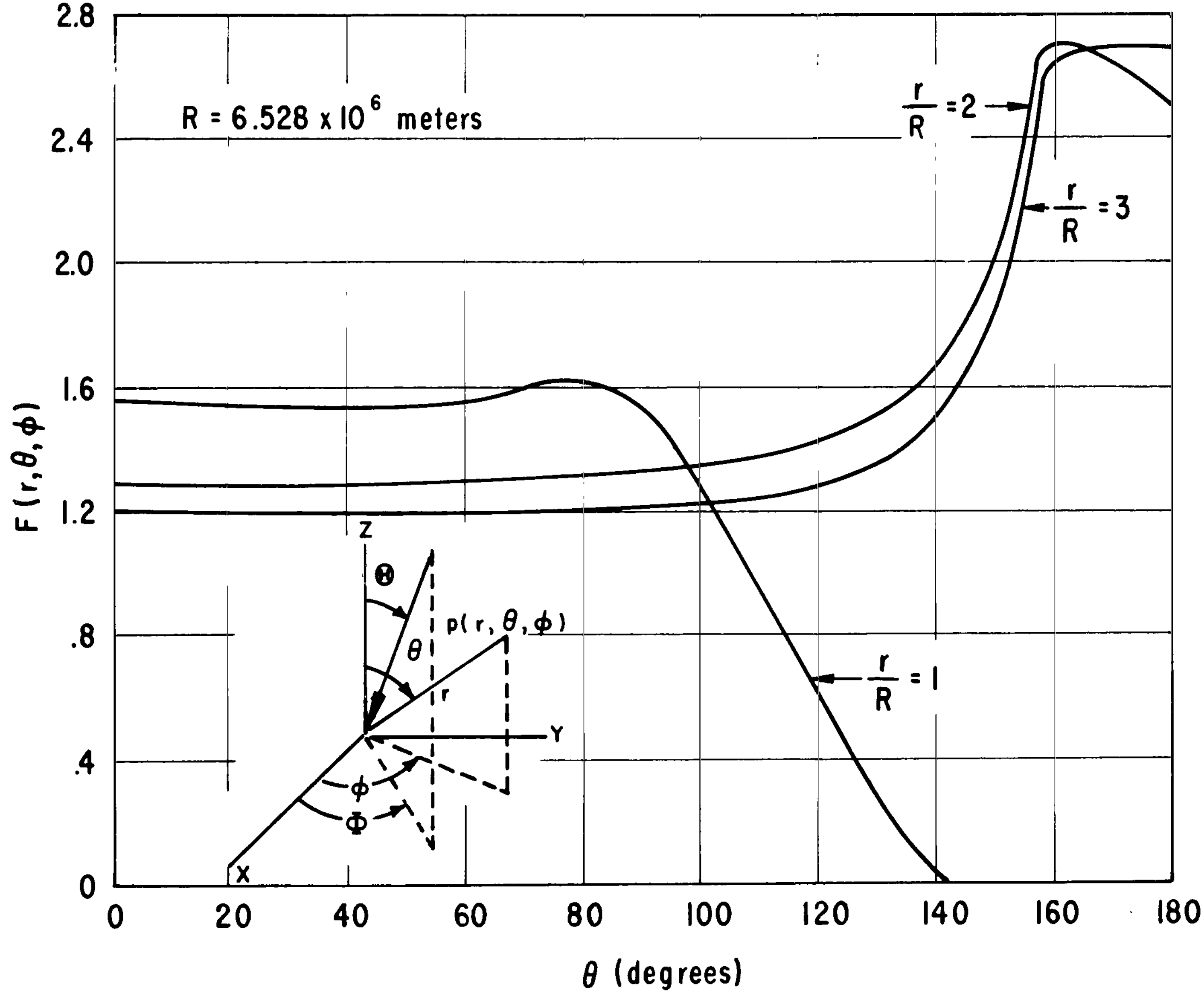


FIGURE 5. METEOROID FLUX FOR THE LIMITED POLAR ISOTROPIC CASE WITH $v_\infty = 10$ km/sec, $0° \leq \Theta \leq 30°$, $0° \leq \Phi \leq 2\pi$, AND $\phi = 0°$, ASSUMING UNIT FLUX AT INFINITY.

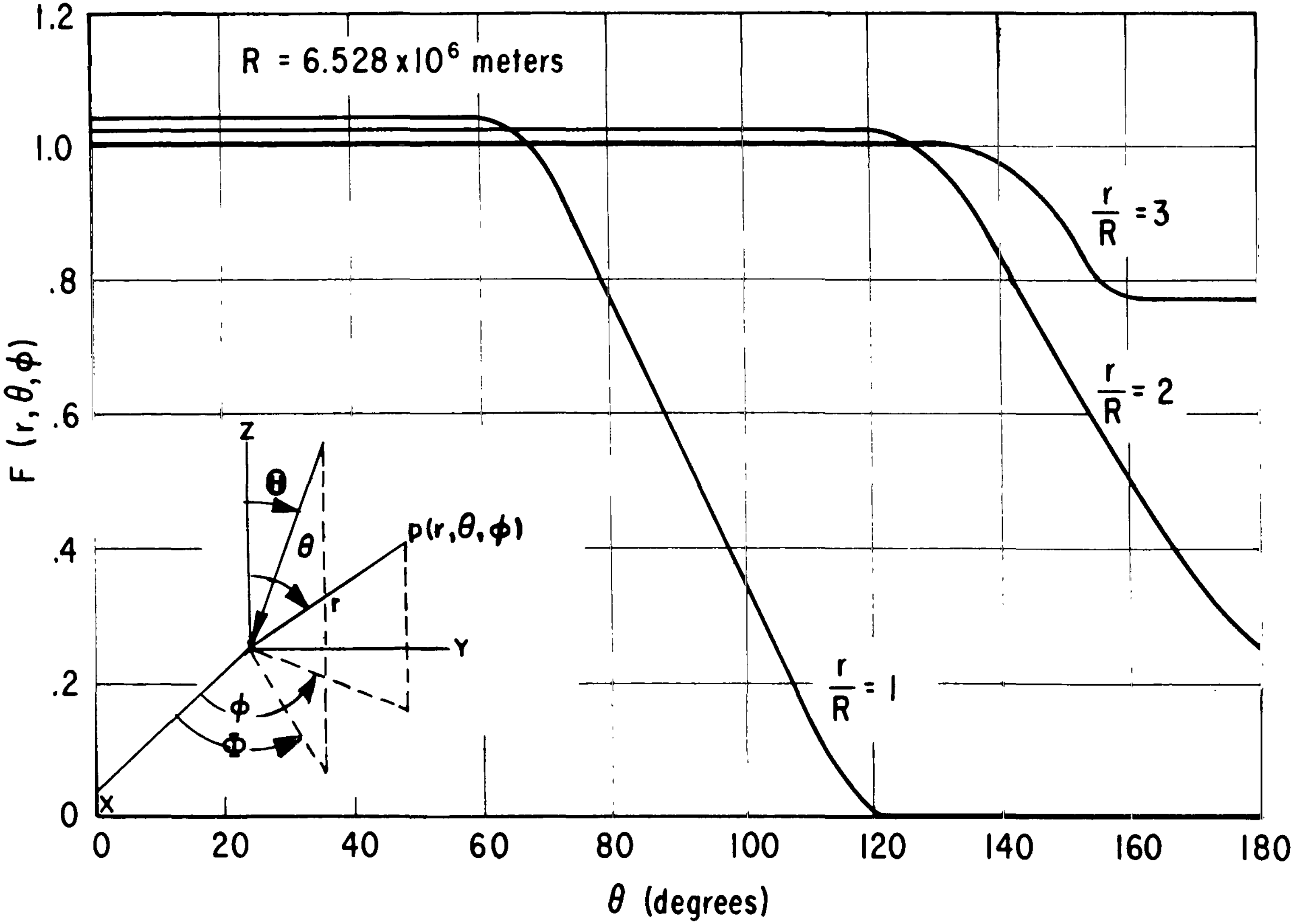


FIGURE 6. METEOROID FLUX FOR THE LIMITED POLAR ISOTROPIC CASE WITH $v_\infty = 40$ km/sec, $0° \leq \Theta \leq 30°$, $0° \leq \Phi \leq 2\pi$, AND $\phi = 0°$, ASSUMING UNIT FLUX AT INFINITY.

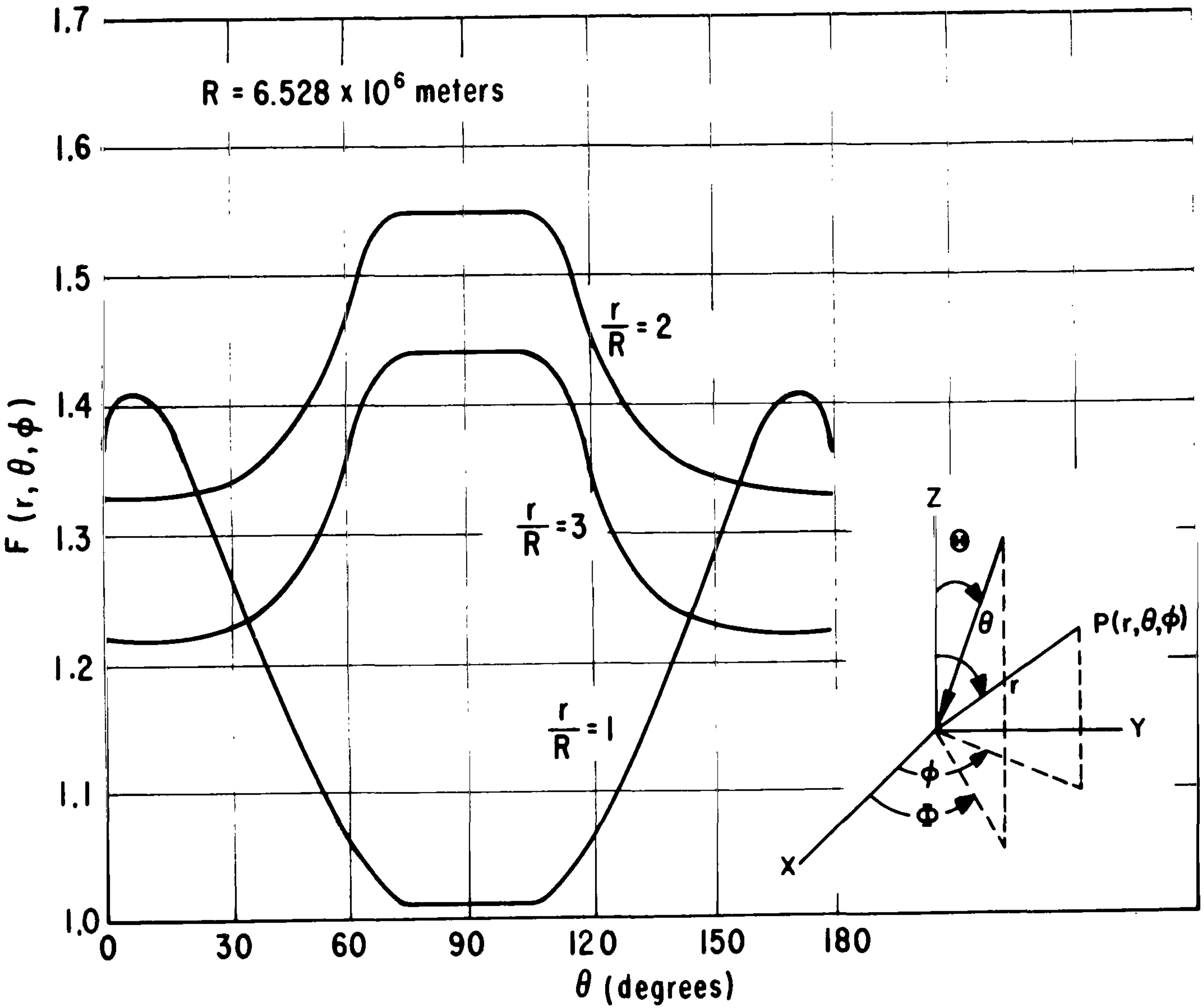


FIGURE 7. METEOROID FLUX FOR THE LIMITED EQUATORIAL ISOTROPIC CASE WITH $v_\infty$ = 10 km/sec, 60° ≤ Θ ≤ 120°, 0° ≤ Φ ≤ 360°, AND $\phi$ = 0°, ASSUMING UNIT FLUX AT INFINITY.

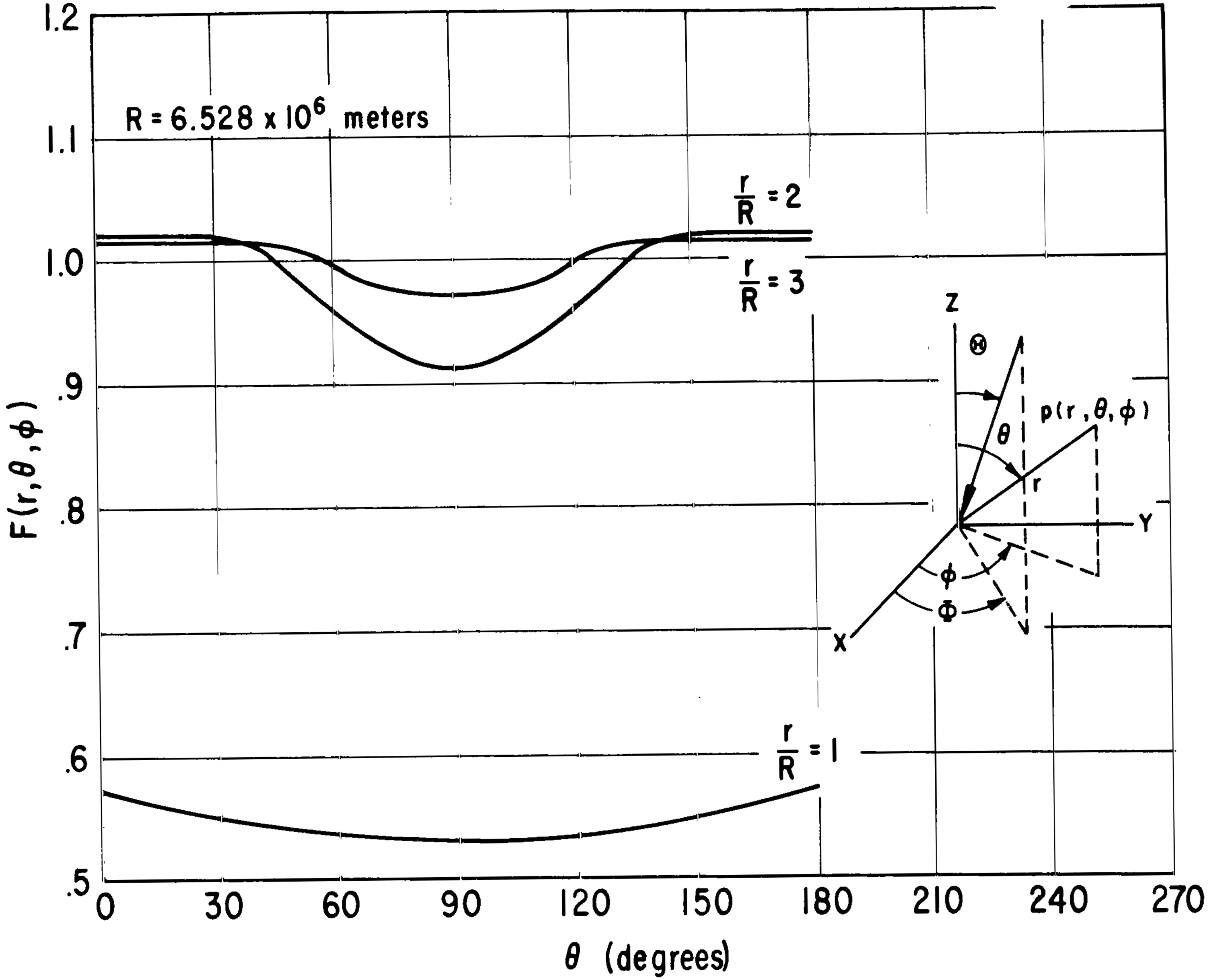


FIGURE 8. METEOROID FLUX FOR THE LIMITED EQUATORIAL ISOTROPIC CASE WITH $v_\infty = 40$ km/sec, $60° \le \Theta \le 120°$, $0° \le \Phi \le 360°$ AND $\phi = 0°$, ASSUMING UNIT FLUX AT INFINITY.

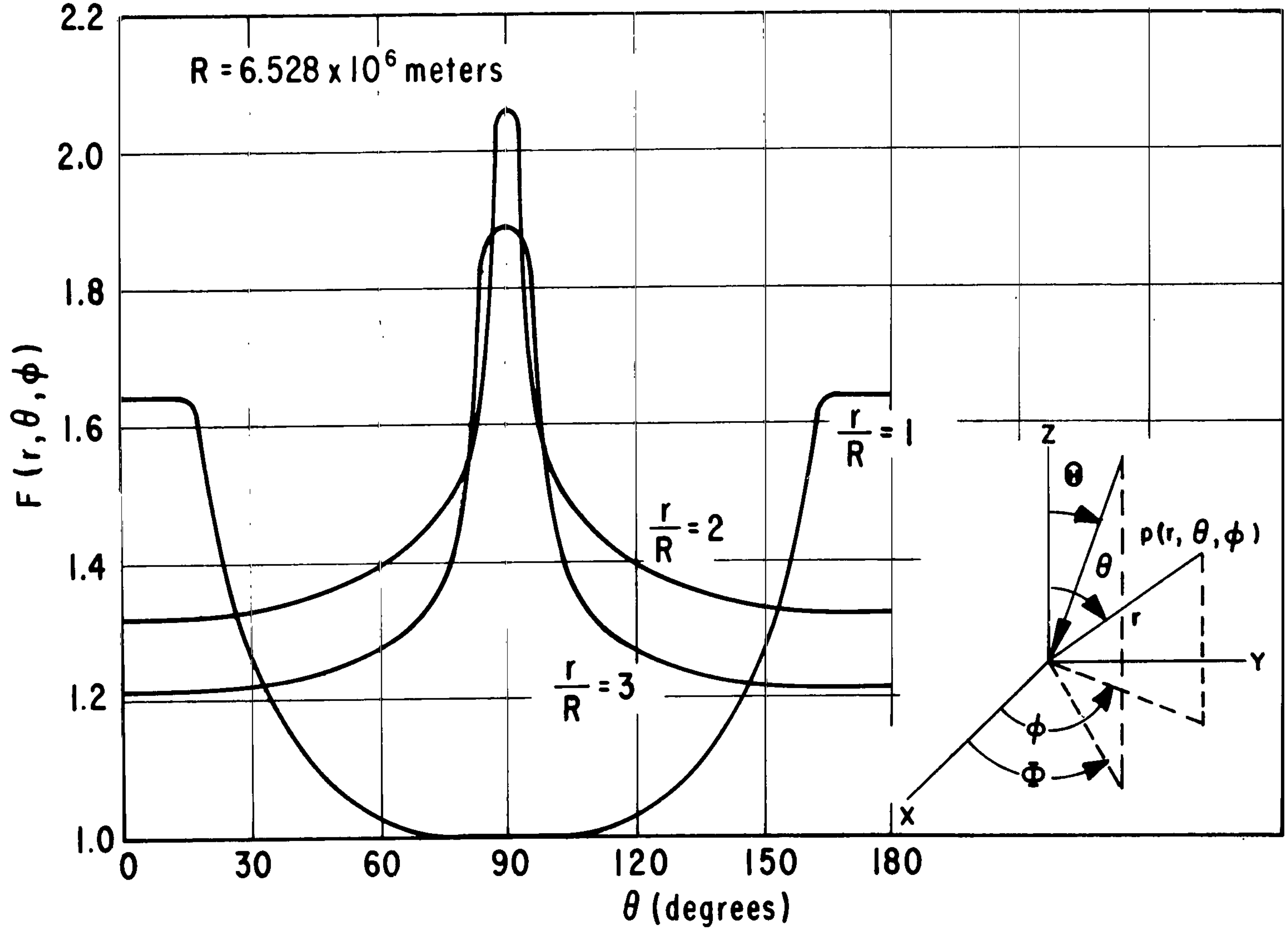


FIGURE 9. METEOROID FLUX FOR THE LIMITED EQUATORIAL ISOTROPIC CASE WITH $v_\infty$ = 10 km/sec, 86°.17 $\leq \Theta \leq$ 93°.83, 0° $\leq \Phi \leq$ 360° AND $\phi$ = 0°, ASSUMING UNIT FLUX AT INFINITY.

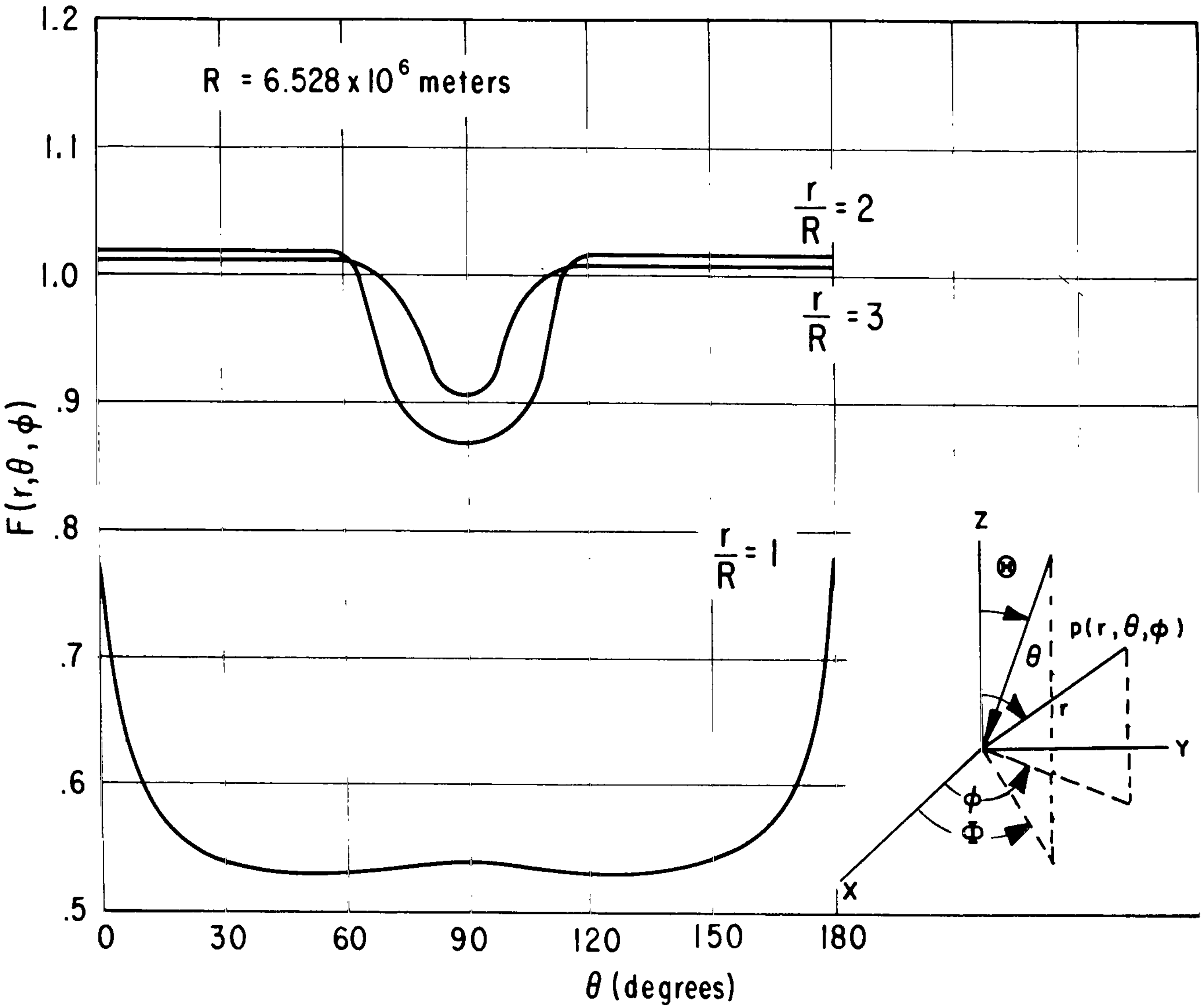


FIGURE 10. METEOROID FLUX FOR THE LIMITED EQUATORIAL ISOTROPIC CASE WITH $v_{\infty}$ = 40 km/sec, $86°17 \leq \Theta \leq 93°83$, $0° \leq \Phi \leq 360°$ AND $\phi = 0°$, ASSUMING UNIT FLUX AT INFINITY.

incidence--for example, compare Figure 3 with Figure 7) reveals profound differences in the flux patterns. In all cases, polar angles are measured from the symmetry axis of the distribution.

## SECTION VI. THE TRANSFORMED ISOTROPIC CASE

In a given coordinate system, a normalized uniform monoenergetic isotropic distribution may be expressed as

$$N(\vec{r}, \vec{v}) = \delta\, [v - v_\infty] / 4\, \pi\, v_\infty^3 \quad \text{(of eq. 19)}. \tag{26}$$

In a second coordinate system containing the Earth and moving with velocity $\vec{V}$ relative to this first system, the distribution has the form

$$N' (\vec{r'}, \vec{v'}) = \delta\, [v\, (v', \mu') - v_\infty] / 4\, \pi\, v_\infty^3 \;, \tag{27}$$

which is the primed analogue of equation 126 in Reference [1], where the variables are related, as shown in Figure 11, by the equation

$$\vec{v} = \vec{v'} + \vec{V}\;. \tag{28}$$

Note that equation 27 is not normalized in the primed (the moving) system. The undisturbed flux F ( ∞) has been computed from the distribution of equation 27 to be

$$F(\infty) = \begin{cases} 1 + \left(\frac{1}{3}\right) \left(\frac{V}{v_\infty}\right)^2 & \text{if } V < v_\infty \\ \left(\frac{V}{v_\infty}\right) \left[1 + \frac{1}{3} \left(\frac{v_\infty}{V}\right)^2\right] & \text{if } V > v_\infty. \end{cases} \tag{29}$$

(Refer to eqs. 138 and 139 in Ref. [1].)

The flux near a disturbing center can be written, by virtue of equations 1, 27, and 29 as

$$F(r, \theta, \phi) = \iiint \frac{\delta\, [\, v(v', \mu') - v_\infty]}{4\, \pi\, v_\infty^3\, F\, (\infty)}\, f(v', r, \theta')\, v'^3\, dv'\, d\mu'\, d\phi', \tag{30}$$

where one should note the division by $F(\infty)$ required for normalization in the moving system, and realize that r, $\theta$ in $F(r, \theta, \phi)$ now refer to the moving system. In order to correctly perform the $\delta$ function integration, we need to find the differential of v $(v', \mu')$. From equation 28 and Figure 11

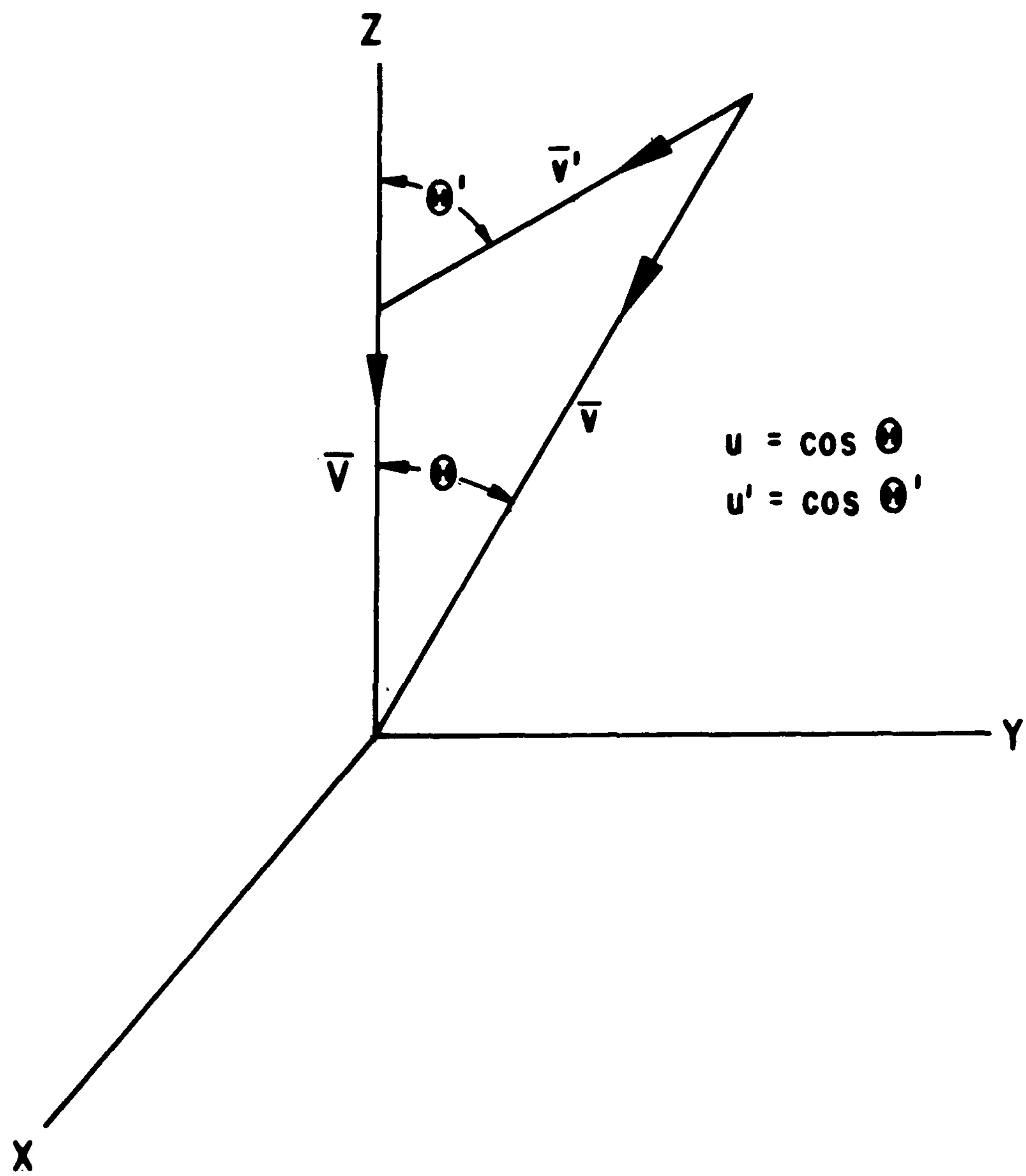


FIGURE 11. THE VELOCITY ADDITION LAW

$$v = (v'^2 + V^2 + 2v' V \mu')^{\frac{1}{2}} \quad \text{and} \tag{31}$$

$$dv = (v' + V\mu')\, dv' / (v'^2 + V^2 + 2v' V\mu')^{\frac{1}{2}} . \tag{32}$$

Equation 31 is solved for v' by the quadratic formula to eliminate v' from equation 32, obtaining

$$v' = -V \mu' \pm (V^2 \mu'^2 + v^2 - V^2)^{\frac{1}{2}} . \tag{33}$$

Substituting into equation 32, one finds

$$dv \approx \frac{(V^2 \mu'^2 + v^2 - V^2)^{\frac{1}{2}}}{v} \, dv' . \tag{34}$$

Using this information, with $v = v_\infty$, the integral of equation 30 can be reduced to

$$F(r, \theta, \phi) = \frac{1}{4 \pi v_\infty^3 \, F(\infty)} \int\int \frac{v_\infty \, f(v', r, \theta') \, v'^3_\infty \, d\mu' \, d\phi'}{(V^2 \mu'^2 + v_\infty^2 - V^2)^{\frac{1}{2}}} , \tag{35}$$

where $\vec{v}'_\infty \equiv \vec{v}' \, (\vec{v}_\infty)$. At large distances $f(v_\infty, r, \theta') \rightarrow 1$ and the integral over $\phi'$ yields $2\pi$; thus the undisturbed flux is given by

$$F(\infty) = \frac{1}{2 v_\infty^2} \int \frac{[-V\mu' \pm (V^2 \mu'^2 + v_\infty^2 - V^2)^{\frac{1}{2}}]^3}{(V^2 \mu'^2 + v_\infty^2 - V^2)^{\frac{1}{2}}} \, d\mu', \tag{36}$$

which reduces to equation 29. For $V < v_\infty$, only the plus sign corresponds to physical reality and $\mu'$ is integrated between the limits of -1 and +1. For $V > v_\infty$, an integration for both signs is necessary over the range $-1 < \mu' < -\sqrt{1 - (v_\infty/V)^2}$. Only this range of $\mu'$ yields real positive v' values. The minus sign for every $\mu'$ provides the contribution from the forward hemisphere of directions ($\mu > 0$), and the plus sign the contribution from the aft hemisphere ($\mu < 0$) to the distribution in the primed system.

By dividing the flux integral by $F(\infty)$, we have required equation 35 to approach unity at large distances. Furthermore, $F(r, \theta, \phi)$ must approach the parallel stream case as $V/v_\infty$ becomes large and the pure isotropic case as $V/v_\infty$ becomes small.

Figures 12 through 15 show the transformed isotropic cases for $v_\infty = 10$ km/sec, $V = 30$ km/sec; $v_\infty = 40$ km/sec, $V = 30$ km/sec; $V = 10$ km/sec, $v_\infty = 40$ km/sec; and $V = 60$ km/sec, $v_\infty = 10$ km/sec.

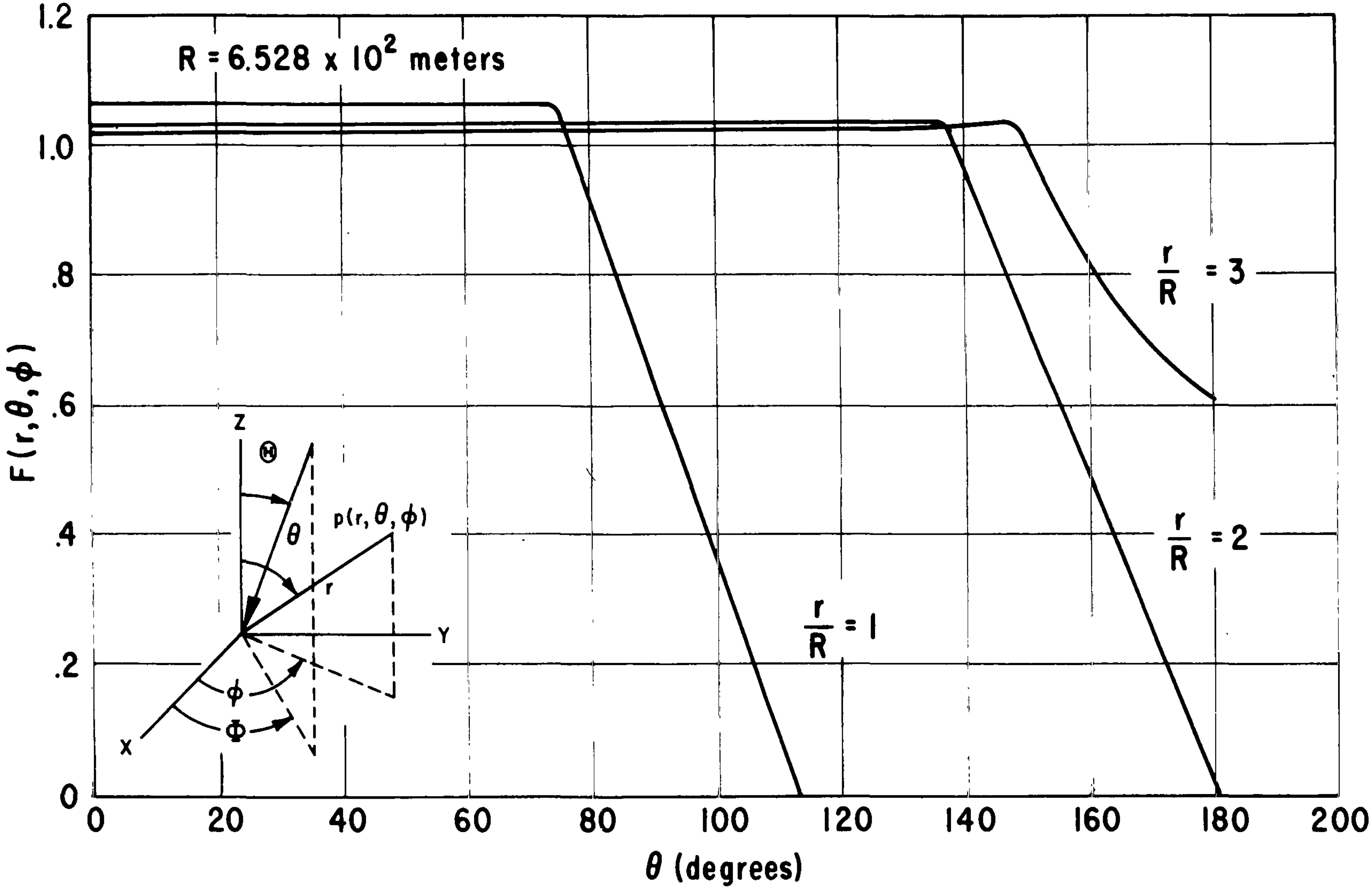


FIGURE 12. METEOROID FLUX FOR TRANSFORMED ISOTROPIC CASE WITH $v_\infty$ = 10 km/sec, 0° ≤ Θ ≤ π, 0° ≤ Φ ≤ 2 π, AND φ = 0°, ASSUMING UNIT APPARENT FLUX AT INFINITY FOR A DETECTOR MOVING AT 30 km/sec IN POSITIVE Z DIRECTION.

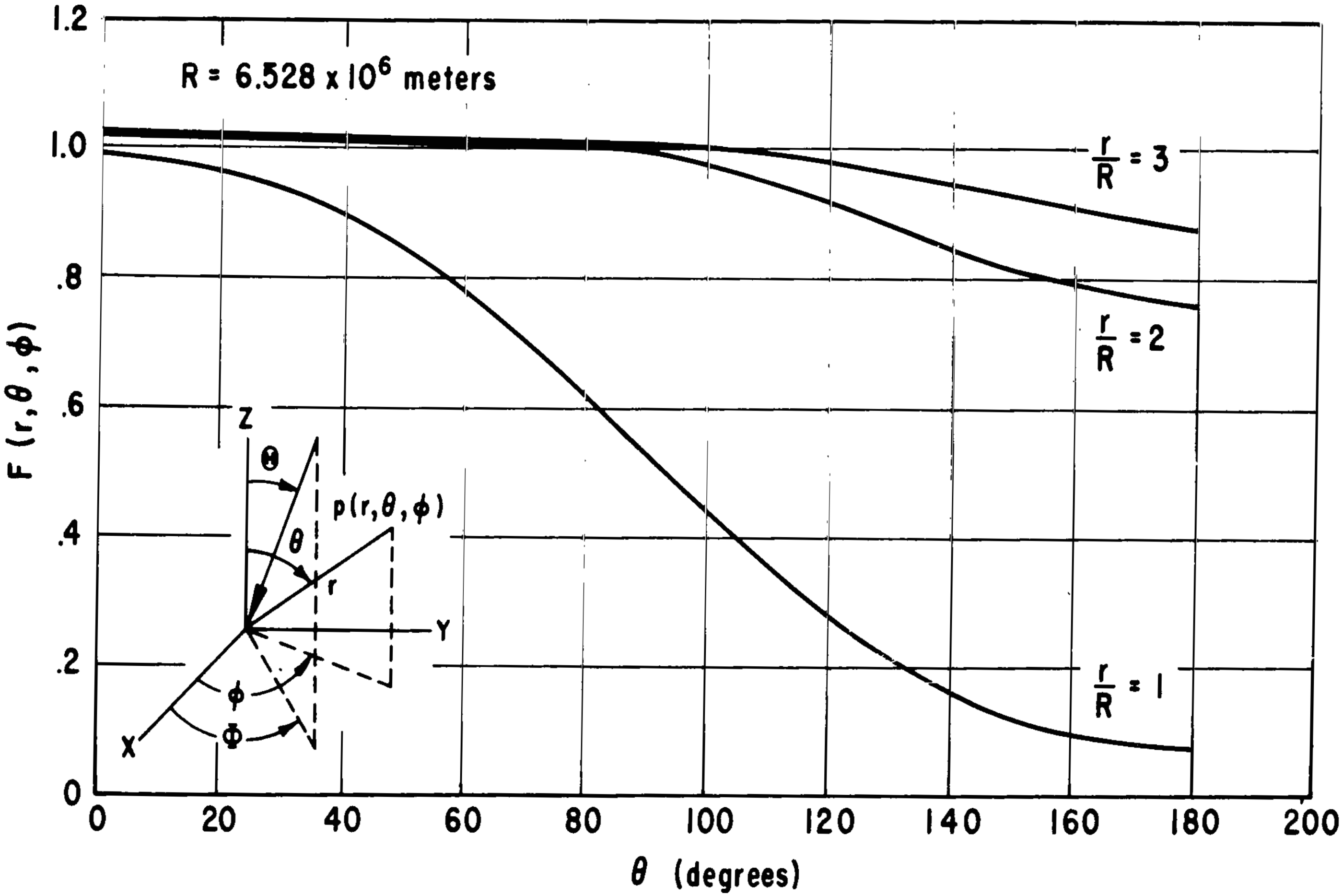


FIGURE 13. METEOROID FLUX FOR TRANSFORMED ISOTROPIC CASE WITH $v_{\infty}$ = 40 km/sec, 0° $\leq \Theta \leq \pi$, 0° $\leq \Phi \leq 2\pi$ AND $\phi$ = 0°, ASSUMING UNIT APPARENT FLUX AT INFINITY FOR A DETECTOR MOVING AT 30 km/sec IN POSITIVE Z DIRECTION

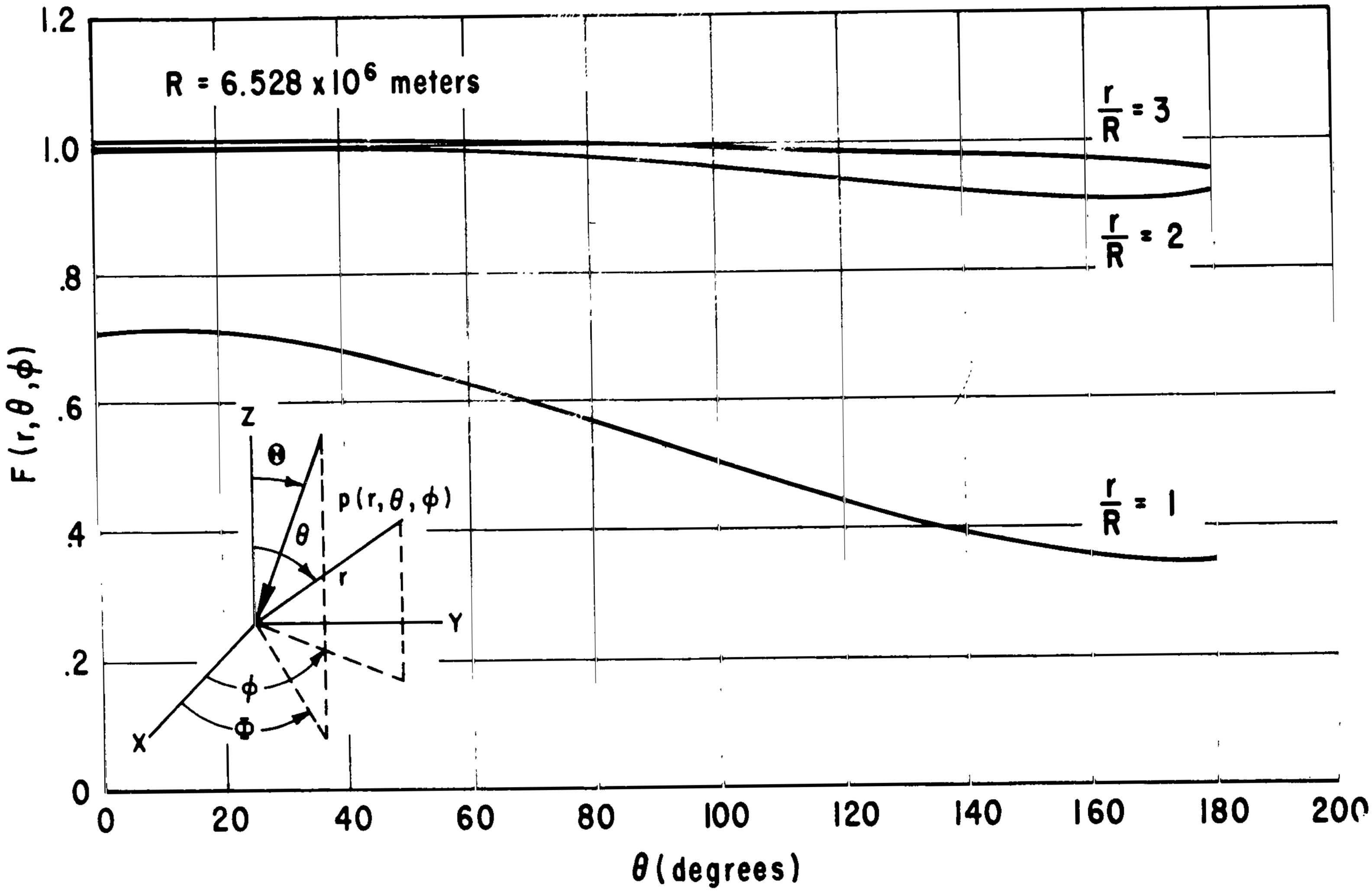


FIGURE 14. METEOROID FLUX FOR TRANSFORMED ISOTROPIC CASE WITH $v_\infty = 40$ km/sec, $0° \leq \Theta \leq \pi$, $0° \leq \Phi \leq 2\pi$ AND $\phi = 0°$, ASSUMING UNIT APPARENT FLUX AT INFINITY FOR A DETECTOR MOVING AT 10 km/sec IN POSITIVE Z DIRECTION

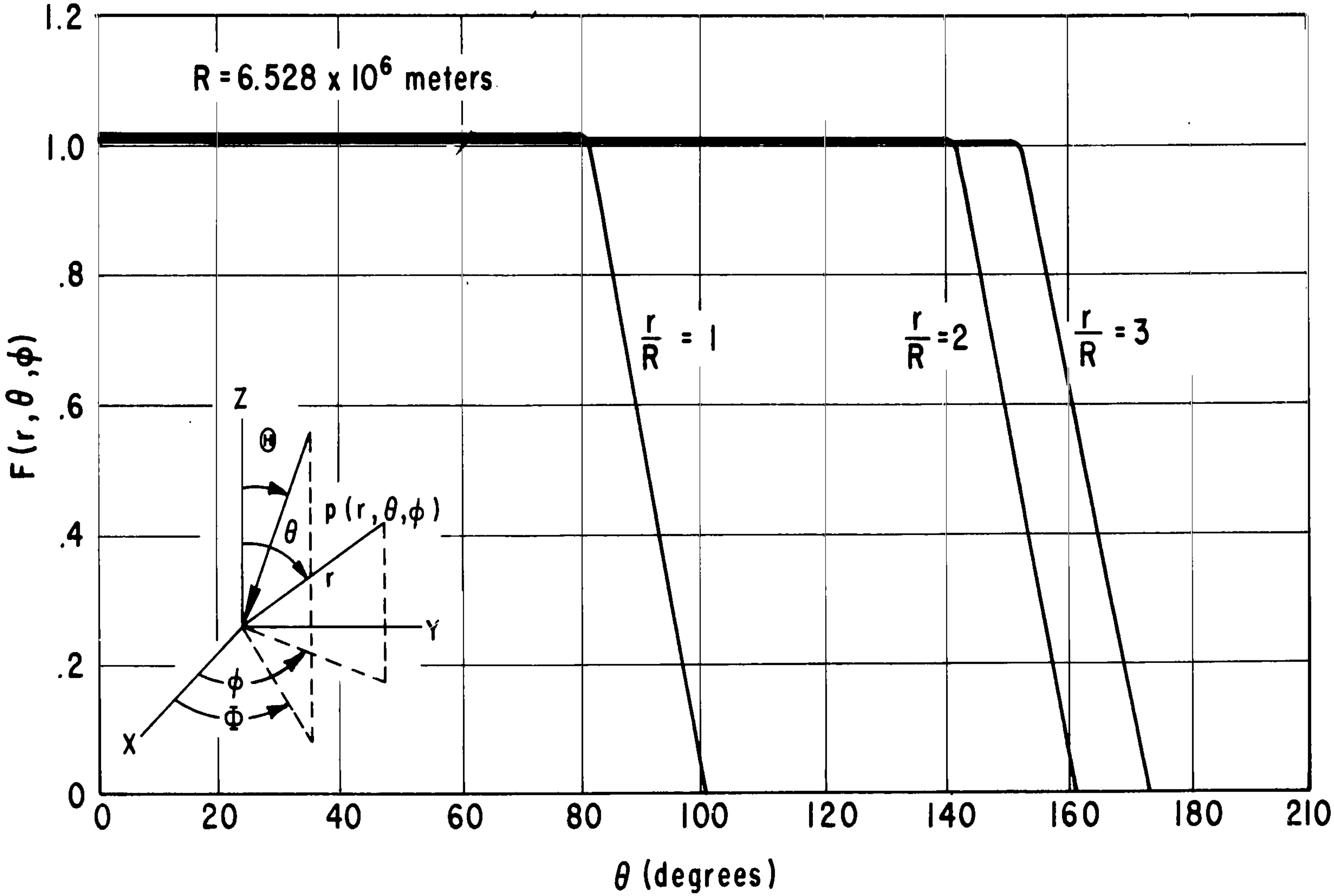


FIGURE 15. METEOROID FLUX FOR THE TRANSFORMED ISOTROPIC CASE WITH $v_\infty = 10$ km/sec, $0° \leq \Theta \leq \pi$, $0° \leq \Phi \leq 2\pi$ AND $\phi = 0°$, ASSUMING UNIT APPARENT FLUX AT INFINITY FOR A DETECTOR MOVING AT 60 km/sec IN POSITIVE Z DIRECTION

A discussion of the computation method used in obtaining these results is given in an appendix.

## SECTION VII. CONCLUSIONS

These calculations, based on rather simple assumptions for the distribution of meteoroids with respect to direction and energy, show that there are not likely to be any sensational focussing effects. However, highly anisotropic distributions result in radial and angular dependences which should be easily observable, either by radar or optical studies of impacts on the atmosphere, or by a large micrometeoroid satellite in a polar orbit, or in a reasonably eccentric orbit.

The transformed isotropic case is interesting in that it can be used to predict flux distributions in the solar system for the case in which the sun moves through a cloud of particles which have some velocity distribution relative to their center of mass. In the limit as the particle speed in their center of mass system goes to zero, the transformed isotropic distribution approaches that of the parallel stream.

## APPENDIX

### Numerical Calculations

The numerical evaluation of the integrals in equations 35 and 36 poses special problems in some cases. In particular, when $v < V$, one obtains

$$F(r, \theta, \phi) = \frac{1}{2\pi v_\infty^2 F(\infty)} \int_{\mu' = -1}^{-\sqrt{1-\left(\frac{v_\infty}{V}\right)^2}} \int_{\Phi = 0}^{\pi} \frac{f(v'_\infty, r, \theta')[-V\mu' \pm (V^2\mu'^2 + v_\infty^2 - V^2)^{\frac{1}{2}}]^3 \, d\mu' d\Phi}{(V^2 \mu'^2 + v_\infty^2 - V^2)^{\frac{1}{2}}}, \quad \text{(A1)}$$

and $F(\infty)$, the undisturbed flux at large distances, is computed by letting $f \equiv 1$. Since all problems considered in this paper possess azimuthal symmetry, the range of the $\Phi$-integration can be taken as 0 to $\pi$ and the value of the integral doubled, as shown. In each case, the integral is to be performed twice, once with the positive sign and once with the negative sign in the integrand; and the two results are to be summed.

The integral in equation A1 is improper since the denominator of the integrand is zero when $\mu'$ attains the value of its upper limit. In order to estimate the value of this integral, first note that equation A1 can be written in the simplified notation

$$F = k_o \int_{\mu = -1}^{-a} \int_{\Phi = 0}^{\pi} \frac{f \cdot [-\mu \pm (\mu^2 - a^2)^{\frac{1}{2}}]^3}{(\mu^2 - a^2)^{\frac{1}{2}}} \, d\mu \, d\Phi \quad \text{(A2)}$$

or

$$F = k \int_{\mu = -1}^{-a} \frac{g(\mu)[-\mu \pm (\mu^2 - a^2)^{\frac{1}{2}}]^3}{(\mu^2 - a^2)^{\frac{1}{2}}} \, d\mu \quad \text{(A3)}$$

and

$$a^2 = \frac{v_\infty^2 - V^2}{V^2}$$

where the dependence on $r$, $\theta$ has been suppressed, the prime has been dropped from $\mu'$, and the dependence on $\mu$ has been expressed by the function g.

Each integral in equation A3 can be replaced by the sum of two integrals, neither of which is improper,

$$\int_{\mu=-1}^{-a} \frac{g(\mu)\left[-\mu \pm (\mu^2 - a^2)^{\frac{1}{2}}\right]^3}{(\mu^2 - a^2)^{\frac{1}{2}}} d\mu = \int_{\mu=-1}^{-(a+\epsilon)} \frac{g(\mu)\left[-\mu \pm (\mu^2 - a^2)^{\frac{1}{2}}\right]^3}{(\mu^2 - a^2)^{\frac{1}{2}}} d\mu$$

$$+ Q \int_{\mu=-(a+\epsilon)}^{-a} g(\mu)\left[-\mu \pm (\mu^2 - a^2)^{\frac{1}{2}}\right]^3 d\mu \quad , \tag{A4}$$

where

$$Q = \frac{\displaystyle\int_{\mu=-(a+\epsilon)}^{-a} \frac{g(\mu)\left[-\mu \pm (\mu^2 - a^2)^{\frac{1}{2}}\right]^3}{(\mu^2 - a^2)^{\frac{1}{2}}} d\mu}{\displaystyle\int_{\mu=-(a+\epsilon)}^{-a} g(\mu)\left[-\mu \pm (\mu^2 - a^2)^{\frac{1}{2}}\right]^3 d\mu} \tag{A5}$$

For sufficiently small $\epsilon$, the function g can be assumed not to vary significantly over the range $-(a+\epsilon)$ to $-a$; that is, $g(\mu)$ can be treated as a constant and cancelled from the expression defining Q.

To estimate the value of Q, the numerator and denominator can now be expanded. Thus, for example,

$$Q_n = -\int_{\mu=-(a+\epsilon)}^{-a} \frac{\mu^3 \, d\mu}{(\mu^2 - a^2)^{\frac{1}{2}}} \pm 3 \int_{\mu=-(a+\epsilon)}^{-a} \mu^2 \, d\mu - 3 \int_{\mu=-(a+\epsilon)}^{-a} \mu \, (\mu^2 - a^2)^{\frac{1}{2}} d\mu$$

$$\pm \int_{\mu=-(a+\epsilon)}^{-a} (\mu^2 - a^2) \, d\mu \quad , \tag{A6}$$

where $Q_n$ is the numerator of the right hand side of equation A5, from which $g(\mu)$ has been cancelled.

Each of these integrals can be treated separately; those which cannot be readily integrated in closed form can be approximated in a power series in $\epsilon^{\frac{1}{2}}$. As an example of the latter method of approximation, consider the evaluation of

$$I_1 = -\int_{\mu=-(a+\epsilon)}^{-a} \frac{\mu^3}{(\mu^2 - a^2)^{\frac{1}{2}}} \, d\mu \ .$$

Let

$$\lambda^2 = \mu^2 - a^2$$

$$\lambda \, d\lambda = \mu \, d\mu \, .$$

Then

$$I_1 = \int_0^{(2a\epsilon+\epsilon^2)^{\frac{1}{2}}} \frac{(\lambda^2 + a^2)}{\lambda} \, \lambda \, d\lambda \tag{A7}$$

$$= \left[ \frac{\lambda^3}{3} + a^2 \lambda \right]_0^{(2a\epsilon+\epsilon^2)^{\frac{1}{2}}} \tag{A8}$$

$$= \frac{1}{3} (2a\epsilon+\epsilon^2)^{\frac{3}{2}} + a^2 (2a\epsilon+\epsilon^2)^{\frac{1}{2}} \tag{A9}$$

$$= \frac{1}{3} (2a\epsilon)^{\frac{3}{2}} \left(1 + \frac{\epsilon}{2a}\right)^{\frac{3}{2}} + a^2 (2a\epsilon)^{\frac{1}{2}} \left(1 + \frac{\epsilon}{2a}\right)^{\frac{1}{2}} \tag{A10}$$

$$\approx \sqrt{2} \, a^{\frac{5}{2}} \epsilon^{\frac{1}{2}} + \frac{11\sqrt{2}}{12} a^{\frac{3}{2}} \epsilon^{\frac{3}{2}} + \frac{15\sqrt{2}}{32} a^{\frac{1}{2}} \epsilon^{\frac{5}{2}} \ , \tag{A11}$$

where equation A11 is accurate through terms of order $\epsilon^{\frac{5}{2}}$ .

Proceeding in this manner, one can show that

$$Q \approx \frac{1}{V \epsilon^{\frac{1}{2}}} \; \frac{\sqrt{2} \, a^{\frac{5}{2}} \pm 3a^2 \epsilon^{\frac{1}{2}} + \frac{35\sqrt{2}}{12} a^{\frac{3}{2}} \epsilon \pm 4a \epsilon^{\frac{3}{2}} + \frac{63\sqrt{2}}{32} a^{\frac{1}{2}} \epsilon^2}{a^3 \pm 2\sqrt{2} \, a^{\frac{5}{2}} \epsilon^{\frac{1}{2}} + \frac{9a^2}{2} \epsilon \pm \frac{7\sqrt{2}}{2} a^{\frac{3}{2}} \epsilon^{\frac{3}{2}}} \tag{A12}$$

where numerator and denominator are accurate through terms of order $\epsilon^2$.

In order to test the validity of this approach to evaluating the improper integral (A1), the normalized undisturbed flux $F_\infty$ can be computed and the results compared with the known true value of unity. Table A1 summarizes the results of some typical test calculations.

Table A1

Test Calculations of Undisturbed Flux

| Detector Velocity (km/sec) | Velocity at Infinity (km/sec) | a | $\epsilon$ | Computed $F(\infty)$ | Fraction of $F(\infty)$ due to Contribution from $-(a+\epsilon)$ to $-a$ |
|---|---|---|---|---|---|
| 15 | 10 | .74536 | .00264 | .9952 | .0461 |
| 30 | 10 | .94281 | .00719 | .9994 | .3061 |
| 60 | 10 | .98601 | .00299 | .9998 | .4473 |

In evaluating the integrals discussed in this paper on the IBM 7094, trial calculations were first performed with different mesh sizes using Simpson's rule and the Gaussian quadrature method. Since machine running time is essentially a linear function of the number of times the integrand must be evaluated, it is desirable to keep this number as low as possible consistent with required accuracy. It was found that very good results were obtained when the range of each variable of integration was divided into a number of equal subintervals and the value of the integral on each subinterval obtained using a four-point Gaussian routine. In general the $\Phi$-integration, for which the range in every case was 0 to $\pi$, was performed using 8 subintervals (32 mesh points). The $\mu$-integration was performed with the same number for problems involving the full range -1 to 1 and with a number proportionately reduced when the range was restricted.

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